
Andrew Tendai Zhuga\textsuperscript{1}, Benson Munyaradzi\textsuperscript{2}, and Clement Shonhiwa\textsuperscript{3}

\textit{1. Department of Mechatronic Engineering, 2. Department of Production Engineering, 3. Department of Fuels and Energy; School of Engineering Sciences and Technology; Chinhoyi University of Technology, P. Bag 7724, Chinhoyi, ZIMBABWE, Tel. 263-067-22203, Fax 263-067-23726}

E-mail address for correspondence: andrewtzhuga@cut.ac.zw

Key Words:
vertical axis wind turbine, wind energy, thin airfoil optimization, aerodynamic optimization, wind turbine rotor optimization

Abstract
A general aerodynamic optimization method was used to improve the torque characteristics of a multi-blade vertical axis wind turbine. A decomposition, deformation, and reassembly method was developed to accommodate the variable geometry of the blade during the optimization process. The deformation of the grid was accomplished by a modified version of the Transfinite Interpolation (TFI) method. The method is first applied to a single blade of the turbine and yields a 27\% improvement in overall torque. Further analyses were performed on a single blade with a spanwise slot and two-blade configuration with and without the slots and results indicated more than 10\% further improvement in the overall torque with the slots in place.

Two small-scale multi-bladed (3-blades and 5-blades) prototype turbines were built and tested in the low speed wind speed at stream mean velocity of 2.5 m/sec, which correspond, to Reynolds numbers based on cord length of 1.225 X 10\textsuperscript{5}. The experiments were performed in free air stream on raised ground and in a closed room with a 3-speed stand fan. Results show that at the free stream mean velocity of 2.5 ms\textsuperscript{-1}, the turbines were self-starting and the 5-blade turbine could turn a 6V rated bicycle dynamo generating 4.83V of electricity. At increased wind speeds, the turbines still produced electricity without damage. The power coefficients for the optimized blades extend to a tip speed ratio of 1.6.

Notation

\[
P_w \quad - \quad \text{power in the wind [W]},
\]
\[
P_{mw} \quad - \quad \text{minimum power in the wind [W]},
\]
\[
P_t \quad - \quad \text{the turbine rotor power [W]},
\]
1. Introduction

With a growing focus on renewable energy, interest in the design of wind turbines has also been expanding. In today’s market, the horizontal axis wind turbines (HAWTs) is the most common type in use, but vertical axis wind turbines (VAWTs) have certain advantages. A VAWT need not be oriented with respect to wind direction. Because the shaft is vertical, the transmission and generator can be mounted at ground level allowing easier servicing and a lighter weight, lower cost tower.

However, their designs are not as efficient at harvesting wind energy as the HAWT designs. The basic VAWT designs are the Darrieus, which has curved blades, the Giromill, which has straight blades, and the Savonius, which uses scoops to catch the wind. Wind turbines are either lift-type (pulled by the wind) or drag-type (pushed by the wind). HAWT are lift-type and VAWT are drag, except the Darrieus turbine.

In general, VAWT lift driven turbines, have a higher power potential than HAWT, or drag-driven turbines. A generalized wind pattern map of Zimbabwe shows wind speeds exceeding 3.0 ms⁻¹ stretching diagonally across the country, with the rest of the country experiencing moderate to low wind speeds, below 3.0 ms⁻¹. The relatively low wind speeds that occur in most of Zimbabwe are generally considered insufficient to allow for the cost effective use of wind generators for electricity [1].
This report describes the design and evaluation of a low wind speed wind turbine, for stand-alone application use in the Zimbabwean environment.

2.0 Basic Wind Energy Theory

2.1 What is a wind turbine

A wind energy turbine transforms the kinetic energy of the wind into mechanical or if the turbine if coupled to a generator, electrical energy that can be harnessed for practical use. Mechanical energy is most commonly used for pumping water in rural or remote locations. While, wind electric turbines generate electricity which can be used for homes, institutes and businesses.

2.2 Basic Designs

Wind turbines are classified into two general types: horizontal axis and vertical axis. A horizontal axis machine has its blades rotating on an axis parallel to the ground. A vertical axis machine has its blades rotating on an axis perpendicular to the ground. There are a number of available designs for both and each type has certain advantages and disadvantages. However, compared with the horizontal axis type, very few vertical axis machines are available commercially.

3. VAWT Concept Generation

The most complicated and important aspect of the design was the need to design blades that extract as much energy from the wind as possible throughout a wide range of low to moderate wind speeds, be self-starting, durable, quiet and easy to manufacture. From the mode of operation of wind energy harvesting machines (lift and drag modes) two favorable designs were proposed and analysed.

3.1 Transforming blades concept

The first idea proposed was having blades that form an open-ended triangle that would operate as a drag device (Savonius type) and when rotating would transform from a drag device into an airfoil after which, there will have sufficient lift to drive the blades. By doing this, the blades would operate like a lift device (Darrieus type) at high speeds. If two (or more) of these blades mounted on a shaft, a net torque will be produced due to the sharp nose.

3.2 Helical blades concept

The second idea proposed was to have helical shaped blades, attached to the shaft like Tropeskian with teardrop shaped central support bars that would create sufficient drag to naturally turn the turbine in slow winds, and generates lift in high winds. Several small models with different numbers of blades (three and five) were built to test the idea in a rotational test. A touch test was observed to generate significant torque at low speeds. The results from these crude tests were encouraging.
4.0 Wind turbine design calculations

4.1 Design constraints: Input and output

The turbine was expected to produce between 50 and 500 Watts, while operating at wind speeds ranging between 2.0 to +4.0 ms\(^{-1}\). Assuming that the attached generator will produce about 15V in order to trickle charge a battery.

4.2 Wind turbine efficiency calculations

Rotor power efficiencies were calculated using the formula:

\[ \text{Efficiency} = \eta_{st} = \frac{P_t}{P_w} \]  \hspace{1cm} (1)

Where, \( P_w \) is the available power in the wind, which is given by:

\[ P_w = \frac{A \cdot \rho_{air} \cdot v^3}{2} \]  \hspace{1cm} (2)

\( P_t \) is the power of a wind turbine is given by:

\[ P_t = 0.5 \cdot \eta_{st} \cdot \rho_{air} \cdot A \cdot v^3 \]  \hspace{1cm} (3)

The \( \text{tsr} (\lambda) \) is the ratio of the speed at the tip of the wind turbine blade to the wind speed and is given by:

\[ \lambda = \frac{\omega_{blade} \cdot R}{v} \]  \hspace{1cm} (4)

The efficiency in the function of \( \text{tsr} (\lambda) \) was derived from the following curve fit equation for the \( \text{tsr}(\lambda) \) graphs.

<table>
<thead>
<tr>
<th>Ideal Vertical Wind Turbine</th>
<th>0.5 ( \leq \lambda \leq 1.0 )</th>
<th>1.0 ( \leq \lambda \leq 1.5 )</th>
<th>1.5 ( \leq \lambda \leq 2.5 )</th>
<th>2.5 ( \leq \lambda \leq 4.0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{st} ) = 0.196 * ( \lambda ) + 0.23233</td>
<td>( \eta_{st} ) = 0.104 * ( \lambda ) + 0.32433</td>
<td>( \eta_{st} ) = 0.055 * ( \lambda ) + 0.399</td>
<td>( \eta_{st} ) = 0.022 * ( \lambda ) + 0.481</td>
<td></td>
</tr>
</tbody>
</table>
For $\lambda \geq 4.0$, 
\[
\eta_{st} = -0.078369 * \lambda^2 + 0.92146 * \lambda - 2.3532
\]

The mechanical power of the turbine due to the rotation with the wind power that is captured by the turbine is given by:
\[
P_m = 0.5 * I_{shaft} * \omega_{dar}^3 \tag{6}
\]

To calculate the rotational speed, $\omega_{dar}$, we equated this mechanical power to the wind power that is captured by the turbine, i.e.,
\[
0.5 * \eta_{st} * \rho_{air} * A * v^{(3)} = 0.5 * I_{shaft} * \omega_{dar}^3 \tag{6}
\]

where $I_{shaft}$ is the moment of inertia of the rotor about the rotation shaft which is found by the expression:
\[
I_{shaft} = N_B \rho_B (L_B W_B T_B) R^2 + \frac{N_B \rho_B (L_B W_B T_B) (W_B^2 + t_B^2)}{12} \tag{6}
\]

With two equations (5) and (6) and two unknowns, the rotational speed and the wind turbine efficiency can be solved iteratively.

4.3 Aerodynamic Calculations

The torque produced was determined by the use of basic fluid mechanics calculations. The coefficient of drag and lift were determined for the blades. The net torque was then optimized by using a general aerodynamic optimization method. To accommodate the variable geometry of the blade a decomposition, deformation and assembly method was applied. The deformation of the grid was accomplished by a modified version of the transfinite interpolation (TFI) method [2].

To determine the torque produced, the power produced by the turbine was calculated by the use of the following calculation:
\[
P_{dar} = \frac{C_p * R * v^3 * A}{2} \tag{7}
\]

The solidity of the turbine being calculated as follows:
\[
S = \frac{N_B * C}{D} \tag{8}
\]

The rotational speed of the turbine can then be determined:
\[
\omega_{dar} = \frac{2 * \lambda * \omega_{air} * \pi}{60 * R} \tag{9}
\]

The torque produced by the turbine can then be determined from the power and rotational speed, thus:
The results of the calculations are in appendix B. In brief, the dimensions chosen for the turbine are: radius $R=0.9\, \text{m}$, height $H=1.4\, \text{m}$ and chord length $C=0.16\, \text{m}$. For the turbine to initiate rotation, it must produce enough torque to overcome the frictional torque in the system. The frictional torque in the system was estimated to be less than $1\, \text{N-m}$ in the design phase.

It can be seen that the main parameter in determining the torque is the length of the chord. The turbine will produce enough torque in wind speeds of $0.5\, \text{to}\, 1.5\, \text{ms}^{-1}$ to overcome the frictional torque.

Also, it was observed that, the higher the solidity, the higher the power produced and therefore the higher the torque produced. Hence, it may be desirable to have the solidity as high as possible. Generally, rotors with higher solidities run at lower rotational speeds. Therefore optimal solidities have been found to be in the range of $0.267$ to $0.44$. In conclusion, a solidity of $0.4$ was chosen for the design.

### 4.4 Critical Speed Calculations

This turbine has five blades, a solidity of $0.4$, and a diameter of $1.8\, \text{m}$ so, from eqn [4], the chord length should be $0.16\, \text{m}$. It was observed that the turbines produce power at tip speed ratios of $1.6$ and greater. If it is desired to have the turbine start in wind speeds of $1.5\, \text{ms}^{-1}$ then the blades should be moving at $1.6$ times the speed of the wind. The rotational speed, $\omega_{\text{rot}}$ is found to be $42.44\, \text{rpm}$ from [5], using a $\text{tsr}$ of $1.5$, radius of $0.9\, \text{m}$.

### 5.0 Testing, results and discussion

Two VAWT with the suggested design specifications were made. They were subjected to the same laboratory tests using a three speeded pedestal fan to simulate a wind current. The same response variables were noted. For each fan speed, the output voltage from the (permanent magnet generator) PMG was measured, and compared to the calculated value. The $\text{rpm}$ of the turbine shaft and the torque produced was also calculated.

The results obtained are shown in Appendices B and C. The tests show that the response is well acceptable and within predicted limits. From these measurements and calculations for the dimensions chosen, the following conclusions can be made:

- There are many variables within the design of a vertical axis wind turbine (VAWT), which influence its operation and efficiency. These include:
  - The radius $R$ of the blades
  - The height $H$ of the blades
  - The chord length $C$ of the blades
  - The ‘cut-in’ speed and
  - The air/wind velocity
- The turbine will produce enough torque in wind speeds of $0 \, \text{to} \, 1.5\, \text{ms}^{-1}$ to overcome the frictional torque.
- The higher the solidity, the higher the power produced and therefore the higher the torque produced. It is therefore desirable to have as high a solidity as possible so it can operate in as low a wind speed as possible.

### 6. Conclusion

The initial goal of the project was accomplished, as this is a solution to low wind speed areas. The potential of the concept has been proven to function. This design fills the functions required of a self-starting vertical axis wind turbine for use in low wind areas. It represents a new breed of VAWT suitable even for the urban environment.
REFERENCES

APPENDIX A : DESIGN CALCULATIONS IN DETAIL
i. 1. Design specifications and limitations Minimum power to be extracted from the wind $P_{mw} = 75\ W$

ii. Minimum wind speed to extract that power $v_{mw} = 2.5\ m/s^{-1}$

iii. For stability, diameter to height ratio fixed at $\frac{D}{H} = 1.2$, $\leftrightarrow D = 1.2 * H$

iv. From the selected concept, the turbine is elliptically shaped, hence the swept area is $A_{swept} = \pi * D * H \leftrightarrow A_{swept} = 1.2 * \pi * H^2$
2. Performance calculations

\[ P_{mw} = 75 = 0.625 \times 1.2 \times H^2 \times (2.5)^3 \]

\[ 75 = 36.816 \times H^2 \]

Hence \[ H = \sqrt{\frac{75}{36.316}} \]

\[ = \sqrt{2.037} \]

\[ = 1.427m \]

\[ D = 1.2 \times 1.427 \]

\[ = 1.713m \]

Take \[ \Rightarrow H = 1.4m \text{ and } D = 1.8m \]

Hence \[ \frac{D}{H} = \frac{1.8}{1.4} = 1.29 \approx 1.3 \]

Swept area

\[ A_{swept} = \pi \times 1.8 \times 1.4 \]

\[ = 7.92m^2 \]

Actual extracted power at \( v = 2.5\text{ms}^{-1} \) wind speed

\[ P_v = 0.625 \times 7.92 \times v^3 \]

\[ P_v = 0.625 \times 7.92 \times (2.5)^3 \]

\[ = 77.312W \]

Expected power output at various wind speeds maintaining the turbine swept area constant \( A_{swept} = 7.92m^2 \) are tabulated below:

<table>
<thead>
<tr>
<th>( V [\text{ms}^{-1}] )</th>
<th>( P_v = 4.948 \times v^3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>4.948</td>
</tr>
<tr>
<td>1.5</td>
<td>16.700</td>
</tr>
<tr>
<td>2.0</td>
<td>39.584</td>
</tr>
<tr>
<td>2.5</td>
<td>77.313</td>
</tr>
<tr>
<td>3.0</td>
<td>133.596</td>
</tr>
<tr>
<td>3.5</td>
<td>212.146</td>
</tr>
<tr>
<td>4.0</td>
<td>316.672</td>
</tr>
<tr>
<td>5.0</td>
<td>618.500</td>
</tr>
<tr>
<td>6.0</td>
<td>1068.768</td>
</tr>
</tbody>
</table>

At 6.0\text{ms}^{-1} \text{ maximum rating power will be above } 1\text{kW}.\]
Assume $\lambda = 1.6 \rightarrow 1.5 \leq \lambda \leq 2.5$

Then $\eta_{\text{wt}} = 0.055\lambda + 0.399$

$= 0.055 \times 1.6 + 0.399$

$= 0.487 \rightarrow \eta_{\text{wt}} = 48.7\%$

Turbine rotational speed

$$\omega = \frac{\lambda \times v \times 60}{R \times 2\pi} = \frac{1.6 \times 2.5 \times 60}{0.9 \times 2 \times \pi} = 42.44 \text{ rpm}$$

Torque produced

$$T = \frac{P_w}{\omega} = \frac{77.313}{42.44} = 1.82 \text{ N.m}$$

Check / Proof Calculations

$$\lambda = \frac{\omega \times R \times 2\pi}{v \times 60} = \frac{42.44 \times 0.9 \times 2\pi}{2.5 \times 60} = 1.599 \approx 1.6$$

Solidity

For a 3–blade turbine

$$S = \frac{N_b \times C}{D} = \frac{3 \times 0.16}{1.8} = 0.267$$

For a 5–blade turbine

$$S = \frac{N_b \times C}{D} = \frac{5 \times 0.16}{1.8} = 0.444$$

Mechanical Power from the rotation of the turbine
Adopting the following values \( W_B = 0.16; \ N_B = 5; \ L_B = 2.83 \text{m}; \ t_B = 0.024 \text{m} \) and \( \rho_B = 1.8 \times 10^3 \text{kgm}^{-3} \)

From eqn(6) \[ I_{\text{shaft}} = N_B \rho_B (L_B W_B T_B) R^2 + \frac{N_B \rho_B (L_B W_B T_B) (W_B^2 + t_B^2)}{12} \]

Then let \[ I_1 = \frac{N_B \rho_B (L_B W_B T_B) R^2}{12} \]

\[ = \frac{5 \times 1800 \times 2.83 \times 0.16 \times 0.024 \times 0.9^2}{12} \]

\[ = 79.22 \]

and let \[ I_2 = \frac{N_B \rho_B (L_B W_B T_B) (W_B^2 + t_B^2)}{12} \]

\[ = \frac{5 \times 1800 \times 2.83 \times 0.16 \times 0.024 \times (0.16^2 + 0.9^2)}{12} \]

\[ = 0.213 \]

hence \[ I_{\text{shaft}} = I_1 + I_2 = 79.22 + 0.213 \]

\[ = 79.43 \]

\[ P_m = 0.5 \times I_{\text{shaft}} \times \omega^3 \]

\[ = 0.5 \times 79.43 \times 42.44^3 \]

\[ = 1685.505 \text{ W} \]